VIBRATION OF SQUARE PLATES HAVING VARIOUS EDGE CONDITIONS AND LOADINGS USING A REFINED ELEMENT

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The equation of motion of a thin plate subject to in-plane forces and vibrating with a frequency, ω , is expressed by the variational statement

$$\delta\ (U - W - \omega^2\ T) = 0 \qquad (1)$$

where U is the strain energy, $\omega^2 T$ is the complementary kinetic energy of the plate, and W is the potential energy of the applied forces. All three energy terms are functions of the normal deflection, w, of the plate, and in equn. (1) only w is subject to variation.

For finite element formulation

$$U = \Sigma U_e \qquad W = \Sigma W_e \qquad T = \Sigma T_e \qquad (2)$$

and the energy functions of the elements are expressed in terms of integrals of functions of w and its first and second derivatives. The integrals are taken over the total surface area of the elements.

For a triangular element the normal deflection is assumed to be a quintic polynomial [1]

$$w = \sum_{i=1}^{21} a_i\ \xi^{m_i}\ \eta^{n_i} \qquad (3)$$

and the slopes normal to the three edges of the elements are cubic polynomials. This last assumption reduces the 21 unknown coefficients, a_i, in eqn. (3) to 18. These coefficients, a_i, are expressed in terms of the 18 nodal quantities: the normal displacement, its two first and three second derivatives at each of the three nodal points.

In terms of the global coordinate system the three energy terms can be expressed as

$$U_e = \frac{1}{2}\ D\ \{w\}^t\ [R]^t\ [T_2]^t\ [k]\ [T_2]\ [R]\ \{w\}$$

$$W_e = \frac{1}{2}\ \{w\}^t\ [R]^t\ [T_2]^t\ [k_b]\ [T_2]\ [R]\ \{w\} \qquad (4)$$

$$T_e = \frac{1}{2}\ \omega^2\ \rho\ t\ \{w\}^t\ [R]^t\ [T_2]^t\ [m]\ [T_2]\ [R]\ \{w\}$$

where $[k]$ is the stiffness matrix, $[R]$ is the rotation matrix, $[T_2]$ is the transformation matrix, and $[m]$ is the consistent mass matrix, as given in ref. [1]. The buckling stiffness matrix $[k_b]$ is given in reference [2]. The thickness of the plate is t , its density is ρ , and its flexural rigidity is $D = Et^3/12(1-\nu^2)$.

A general computer program was prepared to perform calculations for any combination of free, simply supported and built-in edge conditions, and for in-plane self-equilibrating loadings expressed in the global coordinate system as:

$$N_{xx} = N\ (\alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4\ xy)$$

$$N_{yy} = N\ (\beta_1 + \beta_2 x + \beta_3 y + \beta_4\ xy) \qquad (5)$$

$$N_{xy} = N\ (\gamma_1 - \beta_3 x - \alpha_2 y - .5\beta_2 x^2\ - .5\alpha_4 y^2)$$

Equn. (5) covers a wide variety of uniform and non-uniform loading conditions.

For specified loading and edge conditions the expressions for U_e , W_e and T_e can now be computed, and using the usual procedure of assembly of elemental matrices, the global equations of the complete plate can be obtained.

Equn. (1) can be expressed as

$$([K] - [K_b] - \omega^2 [M]) \{q\} = \{0\} \qquad (6)$$

which is an eigenvalue problem, where [K] is the stiffness matrix, $[K_b]$ is the buckling stiffness matrix, and [M] is the consistent mass matrix of the complete plate, and {q} is the vector of global displacement parameters.

The buckling load intensity, N_{crit}, can be computed by taking $\omega = 0$ in equn. (6), and expressed as

$$N_{crit} = \lambda_b \pi^2 D/L^2 \qquad (7)$$

where L is the edge length of the square plate. The dimensionless constant, λ_b, depends upon the loading and edge conditions.

The frequency of lateral vibration of the plate can also be computed for different ratios of N/N_{crit} using equn. (6), and then

$$\omega^2_{crit} = \lambda_{vb} D/\rho t \, . \, L^4 \qquad (8)$$

The natural frequency of the free vibration of the plate is obtained if $N/N_{crit} = 0$, i.e. $[K_b] = 0$.

Some of the computed results are shown on Figs. (1) and (2) for plates with the following loading conditions.

a) triangular compression:

$$N_x = N(1-y/L); \; N_y = N_{xy} = 0$$

b) pure bending: $N_x = N(1-2y/L)$; $N_y = N_{xy} = 0$

c) constant shearing: $N_{xy} = N$; $N_x = N_y = 0$ (10)

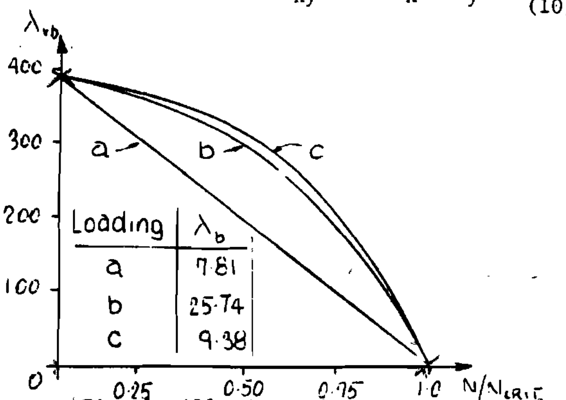

Fig. 1 All edges simply supported

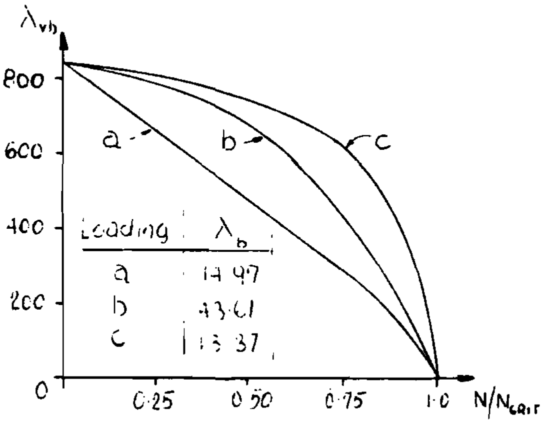

Fig. 2 Edges x = 0 and x = L simply supported, y = 0 and y = L built-in

The use of a refined plate bending element provides an efficient method for computing the frequencies of vibration of plates with different edge conditions and subject to a wide variety of in-plane loadings. The results obtained indicate that the frequency of vibration of plates decreases with the increase of the in-plane forces.

References:

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2. Tabarrok, B., Fenton, R.C., Elsaie, A.M., Application of a Refined Plate Bending Element to Buckling Problems, in print, Int. J. Computers and Structures.